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FINAL REPORT

CHANGING TIME SERIES

ONR CONTRACT N00014-82-K-0292

by

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August 26, 1986

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1. Introduction

The statistical anlaysis of time series data is a challenging and fascinating topic. Much has appeared in the statistical and engineering literature concerning time series analysis, however very few people have attempted the Bayesian approach, but this approach by the principal investigator has been quite successful. Virtually nothing from the Bayesian paradigm had been done until the principal investigator began to investigate this area in the early 1980's. The motivation for using the Bayesian method was to eliminate some of the problems left unresolved by the Box-Jenkins approach.

The objectives of the proposed research were quite ambitious bescause the Bayesian methods of analyzing non-changing time series (such as with the ARMA process) had not been tried, thus the first order of business was to try a Bayesian solution on ARMA processes. Later, processes with time changing parameters were analyzed, thus satisfying the objectives of the proposed research.

2. Accomplishments

Initial stages of research were limited to a study of autoregressive processes. The prior, posterior, and predictive (forecasting) analyses were completed for autoregressive processes and reported in the following articles: Broemeling and Land (1984), Moen, Broemeling, and Salazar (1985), Moen and Broemeling (1985), Salazar (1982), and Broemeling and Shaarawy (1986). The book by Broemeling (1985) contains material developed during this stage of the project. Briefly, the general approach to these problems was to use Bayes theorem to find the posterior and preditive approach. Using a normal-gamma prior distribution, it was shown that the posterior distribution for the autoregressive parametes was a multivariate t and that the one-step ahead forecast distribution was a unvariate t. Thus, one could easily make inferences about the parameter based on standard distributions. The multivariate version of this solution was successfully worked out by Shaarawy (1984).

It can be reported that the Bayesian analysis of AR processes has been accomplished, however the same approach with MA (moving average) and ARMA processes was much more difficult. The moving average process is nonlinear in its parameters, thus there was not a natural conjugate family for the parameters of this model. The approach here was to approximate the likelihood function by estimating the residuals with nonlinear least squares estimates then substituting these estimates into the likelihood function. The approximate likelihood function is then in the normal-gamma form and the posterior, prior, and predictive analysis can be implemented in the same way that one analyzes an autoregressive processes, however now the Bayesian analysis is only an approximation. Shaarawy's (1984) dissertation and later in Broemeling and Shaarawy (1984, 1986), the accuracy of the approximation was determined to be as good as the Box-Jenkins asymptotic method. It was also shown that the four stages of a time series analysis, namely those of identification, estimation, diagnostic checking, and forecasting are more easily implemented from the Bayesian approach. That is to say, the repeated

use of Bayes theorem gives one a complete time series analysis.

The final stage of this contract was to develop the Bayesian analysis of linear dynamic systems (the so-called state space formulation) and to do the same with intervention effect models. Curiously, the Bayesian approach to linear dynamic systems has not been taken during the last two and one-half decades of furious activity, even though Kalman's original solution can be given a Bayesian interpretation. In Broemeling (1985) and in Broemeling, Yusoff, and Diaz (1985) as well as in Broemeling and Diaz (1986), the authors formulated and solved the prediction, filtering, and control problems with linear dynamic systems. In the case of adaptive filtering and control, difficult problems were encountered with finding the posterior distribution of the states of the system. For example, it was found that the joint distribution of the first k states was a k-variate poly t distribution. Using the normal approximation of Broemeling and Abdullah (1984), a sequential analysis of the filtering and control problems was successfully reported on the basis of emperical studies done by the authors.

Most of the studies were confined to the time domain, however one graduate student, namely Cook (1985) was able to do a Bayesian autoregressive spectral analysis. In particular exact small-sample simultaneous confidence bands for the spectral density were constructed and he is generalizing his results to ARMA processes and to multivariate AR models. The advantages of his approach over the classical method are that his method yields exact small-sample bands, compared to the usual asymptolic theory which yield only approximate bands.

The ONR contract supported several graduate students most of which are referenced above. They have made substantial contributions to the field and are continuing to make progress. Before the author began to work in this area, the phrase 'Bayesian time series' was unknown, and it is interesting to note that now it is becoming known to the statistical community. The author and three of his students, all of whom were funded by ONR, have been invited to contribute to a new volume Bayesian Time Series, which will be edited by Spall (1986). The PI has been editor for two special journal issues, one Communications In Statistics (1985) and the other for the Journal of Econometrics (1982. All of these projects were quite relevant to the research supported by ONR.

The principal investigator is continuing to work in time series anlaysis and will now focus his efforts on nonlinear time series, which is the new trend in the field. His latest research efforts will be reported in his forthcoming book Bayesian Time Series Analysis, to be published by Marcel-Dekker.

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